



A polynomial in the form  $a^3 + b^3$  is called a **sum of cubes**.

A polynomial in the form  $a^3 - b^3$  is called a **difference of cubes**.

*Both of these polynomials have similar factored patterns:*

- A sum of cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Diagram illustrating the sign patterns in the factored form of the sum of cubes:

- Arrows point from the box "same sign" to the plus signs in  $(a + b)$  and the plus signs in  $a^2$  and  $b^2$ .
- An arrow points from the box "opposite sign" to the minus sign in  $(a + b)$  and the minus sign in  $-ab$ .
- An arrow points from the text "always +" to the plus sign in  $b^2$ .

- A difference of cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Diagram illustrating the sign patterns in the factored form of the difference of cubes:

- Arrows point from the box "same sign" to the minus signs in  $(a - b)$  and the plus signs in  $a^2$  and  $b^2$ .
- An arrow points from the box "opposite sign" to the plus sign in  $(a - b)$  and the plus sign in  $+ab$ .
- An arrow points from the text "always +" to the plus sign in  $b^2$ .

**Example 1**

Factor  $x^3 + 125$ .

Find the cubic root of 125...

$$\sqrt[3]{125} = 5; 5^3 = 125$$

$$\begin{aligned}x^3 + 125 &= (x)^3 + (5)^3 \\ &= (x + 5) \left[ x^2 - (x)(5) + 5^2 \right] \\ &= (x + 5)(x^2 - 5x + 25)\end{aligned}$$

**Example 2**

Factor  $8x^3 - 27$ .

Find the cubic root of 8 and 27...

$$\sqrt[3]{8} = 2; 2^3 = 8,$$

and

$$\sqrt[3]{27} = 3; 3^3 = 27$$

$$\begin{aligned}8x^3 - 27 &= (2x)^3 - (3)^3 \\ &= (2x - 3) \left[ (2x)^2 + (2x)(3) + 3^2 \right] \\ &= (2x - 3)(4x^2 + 6x + 9)\end{aligned}$$

### Example 3

Factor  $2x^3 + 128y^3$ .

First find the GCF. GCF = 2; factor out 2...

$$\sqrt[3]{64} = 4; 4^3 = 64$$

$$\begin{aligned} 2x^3 + 128y^3 &= 2(x^3 + 64y^3) \\ &= 2[(x)^3 + (4y)^3] \\ &= 2[x + 4y][x^2 - (x)(4y) + (4y)^2] \\ &= 2(x + 4y)(x^2 - 4xy + 16y^2) \end{aligned}$$

### Example 4

Factor  $x^6 - y^6$ .

First, notice that  $x^6 - y^6$  is both a difference of squares and a difference of cubes.

$$x^6 - y^6 = (x^3)^2 - (y^3)^2 \quad x^6 - y^6 = (x^2)^3 - (y^2)^3$$

In general, factor a difference of squares before factoring a difference of cubes.

$$\begin{aligned} x^6 - y^6 &= \underbrace{(x^3)^2 - (y^3)^2}_{\text{difference of squares}} \\ &= \underbrace{(x^3 + y^3)}_{\text{sum of cubes}} \underbrace{(x^3 - y^3)}_{\text{difference of cubes}} \\ &= [(x + y)(x^2 - xy + y^2)][(x - y)(x^2 + xy + y^2)] \\ &= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2) \end{aligned}$$