Sum & Difference of Cubes

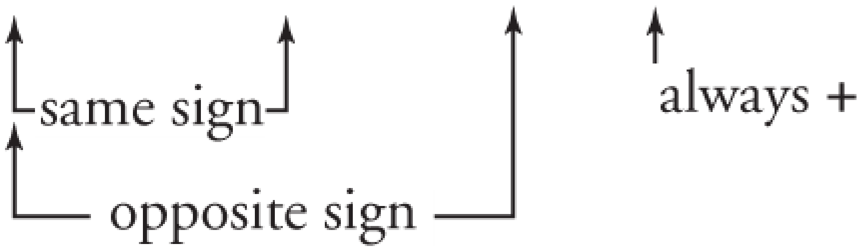
A polynomial in the form $a^3 + b^3$ is called a **sum of cubes.**

A polynomial in the form $a^3 - b^3$ is called a **difference of cubes**.

Both of these polynomials have similar factored patterns:

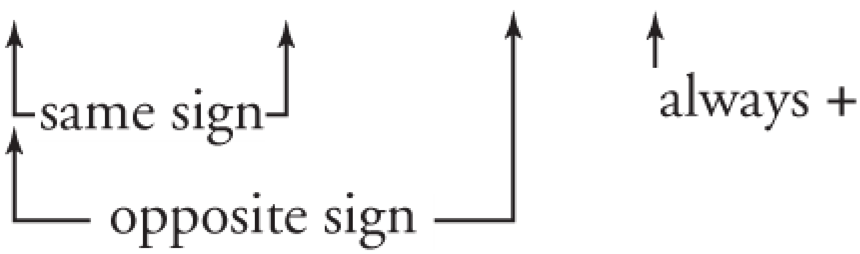
- A sum of cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$



- A difference of cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$



Example 1

Factor $x^3 + 125$.

Find the cubic root of 125...

$\sqrt[3]{125} = 5; 5^3 = 125$

$$\begin{aligned} x^3 + 125 &= (x)^3 + (5)^3 \\ &= (x+5)\left[x^2 - (x)(5) + 5^2\right] \\ &= (x+5)\left(x^2 - 5x + 25\right) \end{aligned}$$

Example 2

Factor $8x^3 - 27$.

Find the cubic root of 8 and 27…

$\sqrt[3]{8} = 2;\ 2^3 = 8,$

and

$\sqrt[3]{27} = 3;\ 3^3 = 27$

$$\begin{aligned} 8x^3 - 27 &= (2x)^3 - (3)^3 \\ &= (2x-3)\left[(2x)^2 + (2x)(3) + 3^2\right] \\ &= (2x-3)\left(4x^2 + 6x + 9\right) \end{aligned}$$

Example 3

Factor 2 x^3 + 128 y^3.

First find the GCF. GCF = 2; factor out 2…

$\sqrt[3]{64} = 4;\ 4^3 = 64$

$$\begin{aligned} 2x^3 + 128y^3 &= 2\left(x^3 + 64y^3\right) \\ &= 2\left[(x)^3 + (4y)^3\right] \\ &= 2[x + 4y]\left[x^2 - (x)(4y) + (4y)^2\right] \\ &= 2(x + 4y)\left(x^2 - 4xy + 16y^2\right) \end{aligned}$$

Example 4

Factor $x^6 - y^6$.

First, notice that $x^6 - y^6$ is both a difference of squares and a difference of cubes.

$$x^6 - y^6 - \left(x^3\right)^2 - \left(y^3\right)^2 \qquad x^6 - y^6 - \left(x^2\right)^3 - \left(y^2\right)^3$$

In general, factor a difference of squares before factoring a difference of cubes.

$$\begin{aligned} x^6 - y^6 &= \underbrace{\left(x^3\right)^2 - \left(y^3\right)^2}_{\text{difference of squares}} \\ &= \underbrace{\left(x^3 + y^3\right)}_{\text{sum of cubes}}\underbrace{\left(x^3 - y^3\right)}_{\text{difference of cubes}} \\ &= \left[(x + y)\left(x^2 - xy + y^2\right)\right]\left[(x - y)\left(x^2 + xy + y^2\right)\right] \\ &= (x + y)\left(x^2 - xy + y^2\right)(x - y)\left(x^2 + xy + y^2\right) \end{aligned}$$