A polynomial in the form $a^3 + b^3$ is called a **sum of cubes**.

A polynomial in the form $a^3 - b^3$ is called a **difference of cubes**.

*Both of these polynomials have similar factored patterns:*

- A sum of cubes:
  \[
  a^3 + b^3 = (a + b)(a^2 - ab + b^2) 
  \]
  - same sign
  - opposite sign
  - always +

- A difference of cubes:
  \[
  a^3 - b^3 = (a - b)(a^2 + ab + b^2) 
  \]
  - same sign
  - opposite sign
  - always +
Example 1

Factor $x^3 + 125$.

Find the cubic root of 125...

$\sqrt[3]{125} = 5; 5^3 = 125$

$x^3 + 125 = (x)^3 + (5)^3$

$= (x + 5)\left[ x^2 - (x)(5) + 5^2 \right]$

$= (x + 5) \left( x^2 - 5x + 25 \right)$

Example 2

Factor $8x^3 - 27$.

Find the cubic root of 8 and 27...

$\sqrt[3]{8} = 2; 2^3 = 8$

and

$\sqrt[3]{27} = 3; 3^3 = 27$

$8x^3 - 27 = (2x)^3 - (3)^3$

$= (2x - 3)\left[ (2x)^2 + (2x)(3) + 3^2 \right]$

$= (2x - 3) \left( 4x^2 + 6x + 9 \right)$
Example 3

Factor \(2x^3 + 128y^3\).

First find the GCF. GCF = 2; factor out 2...

\[\sqrt[3]{64} = 4; \quad 4^3 = 64\]

\[2x^3 + 128y^3 = 2(x^3 + 64y^3)\]

\[= 2\left[(x)^3 + (4y)^3\right]\]

\[= 2[x + 4y]\left[x^2 - (x)(4y) + (4y)^2\right]\]

\[= 2(x + 4y)(x^2 - 4xy + 16y^2)\]

Example 4

Factor \(x^6 - y^6\).

First, notice that \(x^6 - y^6\) is both a difference of squares and a difference of cubes.

\[x^6 - y^6 = (x^3)^2 - (y^3)^2 \quad x^6 - y^6 = (x^2)^3 - (y^2)^3\]

In general, factor a difference of squares before factoring a difference of cubes.

\[x^6 - y^6 = \left(\frac{(x^3)^2 - (y^3)^2}{\text{difference of squares}}\right)\]

\[= \left(\frac{x^3 + y^3}{\text{sum of cubes}}\right)\left(\frac{x^3 - y^3}{\text{difference of cubes}}\right)\]

\[= \left[(x + y)(x^2 - xy + y^2)\right]\left[(x - y)(x^2 + xy + y^2)\right]\]

\[= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)\]